Trading Lecture Notes

Weeks 1 to 4

Contents

1 Week 1: Market Structure and Dual Listings

1.1 Market participants

- Directional participants trade because they expect future price changes. Their objective is to profit from mispricing or information. They are willing to pay transaction costs such as the spread because timing and execution certainty matter.
- Non-directional participants (market makers) do not take views on direction. They provide continuous liquidity by quoting both sides of the market and earn the spread as compensation for taking inventory risk and adverse selection risk.

Key idea: liquidity exists because some participants are willing to hold temporary risk so others can trade immediately.

1.2 Liquidity

Liquidity has two independent components:

- Depth measures how much volume can be traded near the current price before price moves. It reflects the willingness of liquidity providers to hold inventory at different price levels.
- Tightness measures the bid-ask spread. It represents the cost of immediacy, since crossing the spread is effectively paying for instant execution.

A market is only truly liquid when both depth and tightness are strong.

1.3 Limit order book

The limit order book is a priority-based matching system.

$$\text{execution priority} = \text{price priority} + \text{time priority}$$

This structure ensures:

- better prices execute first, which enforces price discovery
- earlier orders at the same price are rewarded, which encourages fair participation

Spread and mid price:

$$\text{Spread} = A - B \tag{1}$$
$$\text{Mid} = \frac{A + B}{2} \tag{2}$$

Mid price is an implied equilibrium estimate assuming symmetric liquidity.

1.4 Why spread exists

The spread compensates liquidity providers for:

- adverse selection risk from informed traders
- inventory risk from holding positions
- operational and capital costs

So:

$$\text{spread} = \text{cost of providing immediacy}$$

1.5 Dual listing arbitrage

Two listings represent identical underlying cash flows:

$$S_1 = S_2 \quad \text{in equilibrium}$$

If they diverge, it creates arbitrage because the two instruments are economically interchangeable.

Mechanism:

- buy the cheaper listing
- sell the more expensive listing
- wait for convergence

This works because ownership can be transferred between venues, forcing long-run price equality.

2 Week 2: Futures and ETFs

2.1 Futures pricing intuition

Futures pricing is a no-arbitrage replication of holding the underlying:

$$F = Se^{r\tau} \tag{3}$$

This arises because:

- buying the asset requires capital today
- capital has an opportunity cost equal to the risk-free rate
- futures defer payment, so they must embed this financing cost

So the futures price is not a prediction, but a financing equivalence.

2.2 Why exponential form appears

$$e^{r\tau}$$

comes from continuous compounding, where interest accrues continuously rather than in discrete steps. This ensures time consistency across different horizons.

2.3 Futures arbitrage

If:

$$F > Se^{r\tau}$$

then:

- sell futures
- buy spot asset
- finance purchase at risk-free rate

If:

$$F < Se^{r\tau}$$

then:

- buy futures
- short spot asset
- invest proceeds at risk-free rate

Key property:

return is locked in at trade initiation

2.4 Cost of carry

$$F = S + \text{carry costs} - \text{carry benefits} \tag{4}$$

This is an accounting identity:

- costs increase forward price because holding is expensive
- benefits reduce forward price because holding generates income

For equities with no dividends:

$$F = Se^{r\tau}$$

2.5 EFP

$$\text{EFP} = F - S$$

This represents the market-implied cost of holding the asset over time, combining interest rates, positioning pressure, and funding conditions.

2.6 ETFs

An ETF represents a tradable claim on a basket of assets.

$$NAV = C + MX \tag{5}$$

where:

- X is index level
- M is exposure per share
- C is cash and operational residuals

This is effectively a linear replication of index exposure.

2.7 Creation and redemption

Authorised participants can convert between ETF shares and the underlying basket:

- creation: deliver basket, receive ETF shares
- redemption: deliver ETF shares, receive basket

This ensures:

$$ETF \approx NAV$$

2.8 ETF arbitrage

If ETF deviates from NAV:

- ETF > NAV: buy basket, create ETF, sell ETF
- ETF < NAV: buy ETF, redeem into basket, sell components

This is a physical arbitrage mechanism that forces convergence.

3 Week 3: Options Pricing

3.1 What an option is

An option transforms linear exposure into a nonlinear payoff:

- stock payoff is linear in S_T
- option payoff is a kinked function of S_T

This nonlinearity is what makes volatility relevant.

3.2 Premium structure

$$\text{Premium} = \text{Intrinsic Value} + \text{Time Value} \tag{6}$$

This separates:

- guaranteed value today
- value of future uncertainty

3.3 Intrinsic value

$$IV_{call} = \max(S - K, 0)$$

This is the payoff if exercised immediately, so it is a lower bound on option value.

3.4 Time value intuition

Time value reflects optionality:

- more time increases dispersion of possible outcomes
- dispersion increases probability of extreme positive payoffs

So time value is essentially the value of exposure to the right tail of the distribution.

At expiry:

$$\text{Time Value} = 0$$

3.5 Volatility

Volatility measures dispersion, not direction.

- historical volatility: realised past dispersion
- implied volatility: market-implied future dispersion

Higher volatility increases option value because it increases probability mass in extreme outcomes.

3.6 Black-Scholes model

$$C = SN(d_1) - Ke^{-r\tau}N(d_2) \tag{7}$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau}$$

Interpretation:

- $SN(d_1)$: expected value of receiving the stock if exercised
- $Ke^{-r\tau}N(d_2)$: expected discounted strike payment
- $N(d_2)$: probability of finishing in the money under risk-neutral measure

So the model is:

$$\text{option price} = \text{expected discounted payoff}$$

3.7 Delta

$$\Delta = \frac{\partial V}{\partial S}$$

This is a local linear approximation of a nonlinear payoff.

Interpretations:

- sensitivity to underlying moves
- probability-weighted exposure to finishing in the money
- hedge ratio required for neutrality

For calls:

$$\Delta = N(d_1)$$

3.8 Market making in options

Market makers:

- estimate theoretical value from model inputs
- quote bid and ask around that value
- earn spread and volatility risk premium

3.9 Delta hedging

After trading options:

$$\Delta_{net} \approx 0$$

Hedging removes directional exposure, leaving:

- volatility exposure
- hedging error from discrete rebalancing

3.10 Put-call parity

$$C - P = S - Ke^{-rT} \tag{8}$$

This exists because both sides replicate the same payoff structure, so any deviation creates arbitrage.

4 Week 4: The Greeks and Trading Options as a Market Maker

4.1 Pricing parameters, recap

- S: spot price of underlying
- K: strike price
- τ: time to expiration
- r: interest rate
- σ: implied volatility

Greeks measure sensitivity of option value V to a change in one parameter, holding the others fixed (dividends ignored).

4.2 The first-order Greeks

$$\Delta = \frac{\partial V}{\partial S} \tag{9}$$

$$\text{vega} = \frac{\partial V}{\partial \sigma} \tag{10}$$

$$\rho = \frac{\partial V}{\partial r} \tag{11}$$

$$\theta = \frac{\partial V}{\partial t} \tag{12}$$

4.2.1 Delta

$$\Delta = \frac{\partial V}{\partial S}$$

Interpretations:

- change in option value due to a change in the underlying
- probability of finishing in the money (by approximation)
- hedge ratio: number of shares of underlying needed to hedge

For calls:

$$\Delta = N(d_1)$$

4.2.2 Vega

Vega is the cash value of one volatility point.

$$P\&L = (\mathrm{vol}_{ask} - \mathrm{vol}_{bid}) \times \mathrm{vega}$$

So a MM tries to buy volatility at a relatively low level and sell it at a higher level (or vice versa).

4.2.3 Theta

Time value runs out of the option when time goes by.

$$\text{Time Value} \to 0 \quad \text{at expiry}$$

Long option: theta cost. Short option: theta income.

4.2.4 Rho

Sensitivity to the interest rate. Smallest impact of the Greeks for short-dated options.

4.3 Gamma

Gamma is a second-order Greek: it describes how delta changes if the underlying changes in price. We use delta to hedge options, so if delta changes, we are no longer (perfectly) hedged.

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}$$

- ATM options have the highest gamma
- long calls and long puts both have positive gamma

4.4 Delta changes: charm and vanna

Delta not only changes due to a change in S, but also if other pricing parameters change:

$$\text{charm} = \frac{\partial}{\partial t}\left(\frac{\partial V}{\partial S}\right) = \frac{\partial^2 V}{\partial t\,\partial S} \tag{13}$$

$$\text{vanna} = \frac{\partial}{\partial \sigma}\left(\frac{\partial V}{\partial S}\right) = \frac{\partial^2 V}{\partial \sigma\,\partial S} \tag{14}$$

4.5 Volatility skew

Implied volatility is not the same across different strikes. Plotting σ against K gives the skew.

- historical volatility: realised past dispersion
- implied volatility: market-implied future dispersion

4.6 What does a market maker do

Based on the parameters S, K, τ, r, σ, the MM calculates the call and put values and shows both bid and ask prices to the market, then waits until someone trades against the quote (buy/sell a call/put).

1. calculate call and put values
2. show bid and ask
3. wait for a trade
4. hedge with the underlying

The price of the underlying is input for the option price calculation. After a trade in the option, a hedge order in the underlying needs to be executed – both low-latency.

4.7 Payoff of hedged calls and puts

- hedged call: $C - \Delta_C \cdot S$
- hedged put: $P + \Delta_P \cdot S$

A hedged call and a hedged put at the same strike combine into a straddle. Indifferent in trading calls and puts – only an opinion on volatility is needed.

4.8 Realized volatility

Option buy trade + hedge trade, kept in position over time: gamma / theta.

At higher prices of S the option gains positive deltas, which can be sold. At lower prices of S the option gains negative deltas, which can be bought.

- positive gamma generates favourable deltas when S changes
- the MM does an additional hedge trade after S moves, to stay flat delta

Applies in the opposite way if the option is sold instead of bought.

4.9 Summary

- market making in options means trading volatility
- a buy trade in an option can be followed by a sell trade – volpoints gained (or lost), which multiplied by vega give a cash value
- if a bought option is kept in position, time value runs out of the option as time goes by
- to compensate (or beat) the loss of time value, a trader re-hedges the delta-imbalance from gamma when the underlying changes
- by executing these gamma-hedges, a trader realizes volatility
- implied volatility is the expectation of volatility between now and expiration; realized volatility is what has been materialized by gamma-hedging
- the above applies in the opposite way if an option is sold instead of bought

5 Core idea

All markets are linked by structural identities:

- prices must satisfy no-arbitrage constraints
- deviations occur due to friction and information delays
- arbitrage forces convergence
- market makers provide continuous liquidity and risk transfer

Core workflow:

- define theoretical value
- compare to market price
- trade deviation
- hedge exposure
- account for costs